Q1: [15 points] Answer the following Questions.

a) [4 points] The first nine digits of the ISBN-10 of a certain book are 013850587. Compute the check digit for that book?

Here

$$\begin{array}{ccccccccc} 0 & 1 & 3 & 8 & 5 & 0 & 5 & 8 & 7 \\ \hline 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

$$\because x_{10} = \sum_{1}^{9} i \cdot x_i \bmod 11 = (1\cdot 0 + 2\cdot 1 + 3\cdot 3 + 4\cdot 8 + 5\cdot 5 + 6\cdot 0 + 7\cdot 5 + 8\cdot 8 + 9\cdot 7) \bmod 11$$

$$= 230 \bmod 11$$

$$= 10$$

$\therefore$ The check digit is "X".

b) [4 points] Encrypt the message CHEAR UP by translating the letters into numbers, applying the encryption function $f(p) = 17p + 22 \pmod{26}$, and then translating the numbers back into letters.

A — 0
B
C
D
E — 4
F
G
H
I
J — 9
K
L
M
N
O — 14
P
Q
R
S
T — 19
U
V
W
X
Y — 24
Z

$\therefore$ CHEAR UP

2, 7, 4, 0, 17, , 20, 15.

For 'C' $\rightarrow f(2) = 17.2 + 22 \pmod{26}$

$= 34 + 22 \pmod{26}$

$= 56 \pmod{26}$

$= 4 \rightarrow$ 'E'

For 'H' $\rightarrow f(7) = 17.7 + 22 \pmod{26}$

$= 141 = 11 \rightarrow$ 'L'

Applying similar computations,

CHEAR UP $\rightarrow$ ELMWZ YR

Q2: [15 points]

Consider the following relations defined on the set of all <u>*natural numbers*</u>:

$R_1 = \{(a, b) \in \mathbb{N}^2 \mid a > b\}$,
$R_2 = \{(a, b) \in \mathbb{N}^2 \mid a \geq b\}$,
$R_3 = \{(a, b) \in \mathbb{N}^2 \mid a < b\}$,
$R_4 = \{(a, b) \in \mathbb{N}^2 \mid a \leq b\}$,
$R_5 = \{(a, b) \in \mathbb{N}^2 \mid a = b\}$ and
$R_6 = \{(a, b) \in \mathbb{N}^2 \mid a \neq b\}$.

Find the following relations:

i. $R_1 - R_2$
ii. $R_1 \cup R_3$
iii. $R_2 \oplus R_4$
iv. $R_5 \circ R_6$
v. $R_1 \circ R_4$

(i) $\because R_1 = \{(a,b) \in \mathbb{N}^2 \mid a > b\}$ & $R_2 = \{(a,b) \in \mathbb{N}^2 \mid a \geq b$

$\Rightarrow R_1 - R_2 = \{(a,b) \in \mathbb{N}^2 \mid a = b\}$.

(ii) $R_1 \cup R_3 = \{(a,b) \in \mathbb{N}^2 \mid a > b \text{ AND } a < b\}$

$\Rightarrow R_1 \cup R_3 = \{(a,b) \in \mathbb{N}^2 \mid a \neq b\}$.

(iii) $R_2 \oplus R_4 = (R_2 \cup R_4) - (R_2 \cap R_4)$

$= \{(a,b) \in \mathbb{N}^2 \mid \text{for all } a, b\}$

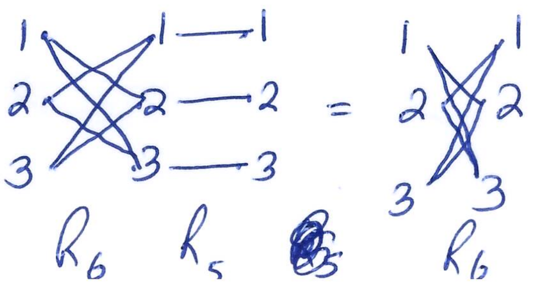
$- \{(a,b) \in \mathbb{N}^2 \mid a = b\}$

$= \{(a,b) \in \mathbb{N}^2 \mid a \neq b\}$.

(iv) $R_5 \circ R_6$ $\therefore$ This is the composition of R_6, R_5

As an example

So $R_5 \circ R_6 = R_6$ since R_5 is the identity relation.



Q4: [20 points]

Let R be the relation on the set of ordered pairs of positive integers such that $((a, b), (c, d)) \in R$ if and only if $ad = bc$.

a) [15 points] Show that R is an equivalence relation.

It is slightly easier to rewrite this relation as:

$R = \{((a,b), (c,d)) \mid \frac{a}{b} = \frac{c}{d}\}$.

Some example pairs here are:

$\{((1,1), (2,2)), ((1,2), (2,4)), ((3,2), (6,4)), \dots\}$

- Reflexive: Yes since $((a,b), (a,b)) \in R$ as, $\frac{a}{b} = \frac{a}{b}$.

- Symmetric: Yes since if $((a,b), (c,d)) \in R$ then $((c,d), (a,b))$ also $\in R$

because, if $((a,b), (c,d)) \in R \Rightarrow \frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{c}{d} = \frac{a}{b} \Rightarrow ((c,d), (a,b)) \in R$.

- Transitive: Yes since if $((a,b), (c,d)) \in R$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$ & if $((c,d), (e,f)) \in R \Rightarrow \frac{c}{d} = \frac{e}{f}$,

then $((a,b), (e,f)) \in R$ since $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$.

$\therefore$ This is an equivalence relation.

b) [5 points] What is the equivalence class of (1, 2) with respect to R?

Equivalence class of (1, 2) is all ordered pairs which have a ratio of (1, 2).

$[(1,2)]_R = \{(1,2), (2,4), (3,6), \dots\}$